SOLUÇÃO PROVA DE FÍSICA

① CÁLCULO DO LIMITE

$$\lim_{x \to 0} \frac{2^x - 1}{x} = \frac{2^0 - 1}{0} = \frac{0}{0}$$

PORTANTO INDETERMINADO. PARA CALCULAR O LIMITE APLICAREMOS A REGRA DE L'HÔPITAL.

$$y(x) = 2^x - 1 \quad , \quad g(x) = x$$

$$\frac{dy(x)}{dx} = \frac{d(2^x)}{dx} \quad , \quad \frac{dg(x)}{dx} = 1$$

$$z = 2^x \therefore \ln z = x \ln 2 \therefore$$

$$\frac{1}{z}\frac{dz}{dx} = \ln 2 \Rightarrow \frac{dz}{dx} = 2^x \ln 2$$

$$\lim_{x \to 0} \frac{2^x - 1}{x} = \lim_{x \to 0} \frac{2^x \ln 2}{1} = \ln 2$$

② $x = a + bt - ct^2$

a.) VELOCIDADE $= v = \frac{dx}{dt} = b - 2ct$

b.) $v = 0 \rightarrow b - 2ct = 0 \rightarrow t = \frac{b}{2c}$

c.) $x = X_{MAX} \longleftrightarrow \frac{dx}{dt} = 0$

ISTO CORRESPONDE AO INSTANTE

$t = \frac{b}{2c}$. PORTANTO,

$$X_{MAX} = x\left(t = \frac{b}{2c}\right) = a + b\left(\frac{b}{2c}\right) - c\left(\frac{b}{2c}\right)^2$$

$$= a + \frac{b^2}{2c} - \frac{b^2}{4c}$$

$$X_{MAX} = a + \frac{b^2}{4c}$$

③ $\int x \ln x \, dx$

$u = \ln x \; ; \quad dv = x \, dx$

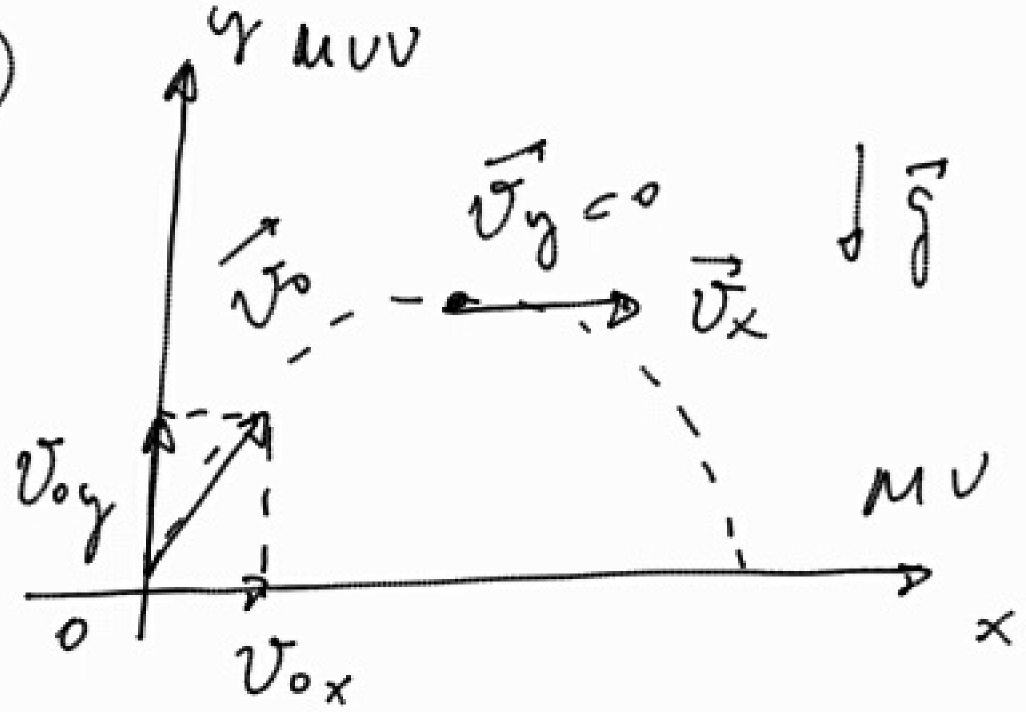
$du = \frac{1}{x} dx \; ; \quad v = \int x \, dx = \frac{x^2}{2}$

$\int x \ln x \, dx = \ln x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x} dx$

$= \frac{x^2}{2} \ln x - \int \frac{x}{2} dx$

$= \boxed{\frac{x^2}{2} \ln x - \frac{x^2}{4} + C}$

(4)



$v_{0x} = v_0 \cos\theta$

$v_{0y} = v_0 \operatorname{sen}\theta$

• DIREÇÃO Ox - MU

$x = (v_0 \cos\theta)t$

• DIREÇÃO Oy - MUV

$y = (v_0 \operatorname{sen}\theta)t - \frac{g}{2}t^2$

$v_y = v_0 \operatorname{sen}\theta - gt$

$v_y^2 = v_0^2 \operatorname{sen}^2\theta - 2gy$

a.) $v_y = v_0 \operatorname{sen}\theta - gt$

$0 = v_0 \operatorname{sen}\theta - gt$

$gt = v_0 \operatorname{sen}\theta$

$\boxed{t = \frac{v_0}{g} \operatorname{sen}\theta}$

b.) $y = (v_0 \operatorname{sen}\theta)t - \frac{g}{2}t^2$

$y = v_0 \operatorname{sen}\theta\left(\frac{v_0}{g}\operatorname{sen}\theta\right) - \frac{g}{2}\left(\frac{v_0^2}{g^2}\operatorname{sen}^2\theta\right)$

$$y = \frac{v_0^2}{g}\,\text{sen}^2\theta - \frac{v_0^2}{2g}\,\text{sen}^2\theta = \boxed{\frac{v_0^2}{2g}\,\text{sen}^2\theta}$$

c.) $\boxed{E_c = \frac{1}{2} m v_0^2 \cos^2\theta}$

d.) $x = (v_0 \cos\theta)\, t$

$$x = v_0 \cos\theta \cdot \frac{2 v_0}{g}\,\text{sen}\,\theta = \boxed{\frac{2 v_0^2}{g}\,\text{sen}\,\theta \cos\theta}$$

e.) $\boxed{E_c = \frac{1}{2} m v_0^2}$

f.) $$\frac{dx}{d\theta} = \frac{d}{d\theta}\left[\frac{2 v_0^2}{g}\,\text{sen}\,\theta\right]\cdot\left[\cos\theta\right] +$$

$$+ \frac{2 v_0^2}{g}\,\text{sen}\,\theta \frac{d}{d\theta}(\cos\theta)$$

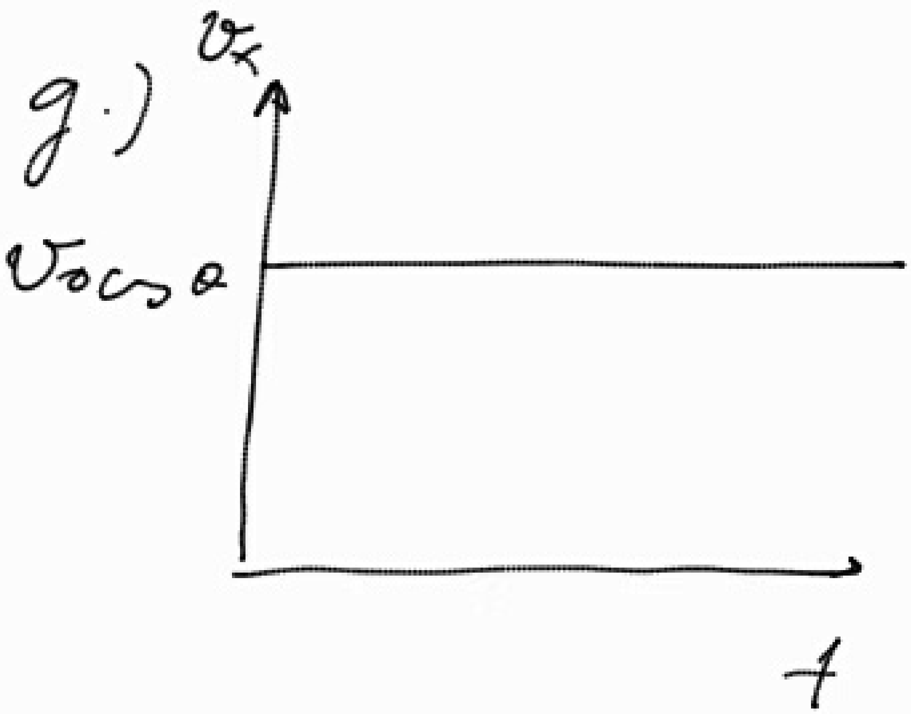
$$= \frac{2 v_0^2}{g}\cos\theta\cos\theta - \frac{2 v_0^2}{g}\,\text{sen}\,\theta\,\text{sen}\,\theta$$

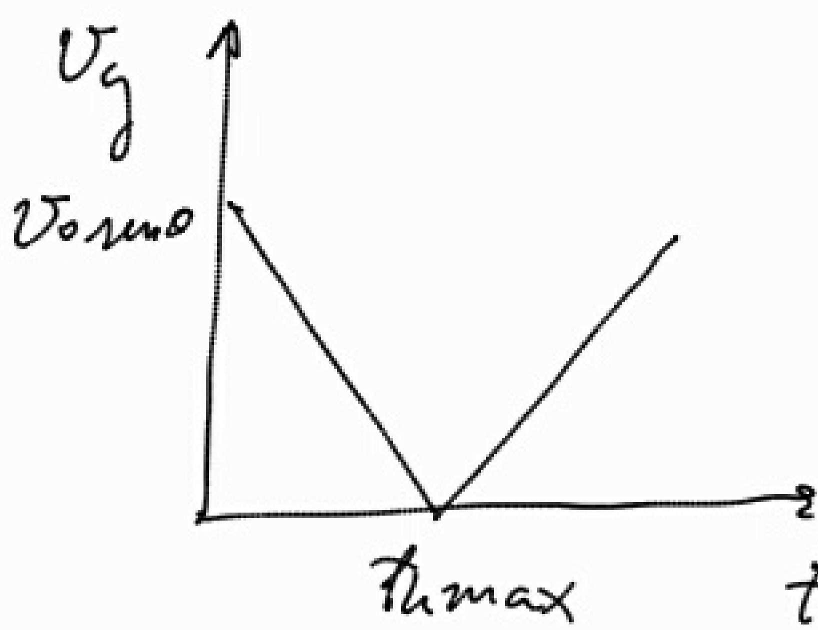
$$= \frac{2v_0^2}{g}\cos^2\theta - \frac{2v_0^2}{g}\,\text{sen}^2\theta$$

$$= \frac{2v_0^2}{g}\left(\cos^2\theta - \text{sen}^2\theta\right) = 0$$

$$\cos^2\theta = \text{sen}^2\theta \rightarrow \cos\theta = \text{sen}\,\theta \rightarrow \boxed{\theta = 45°}$$

g.)





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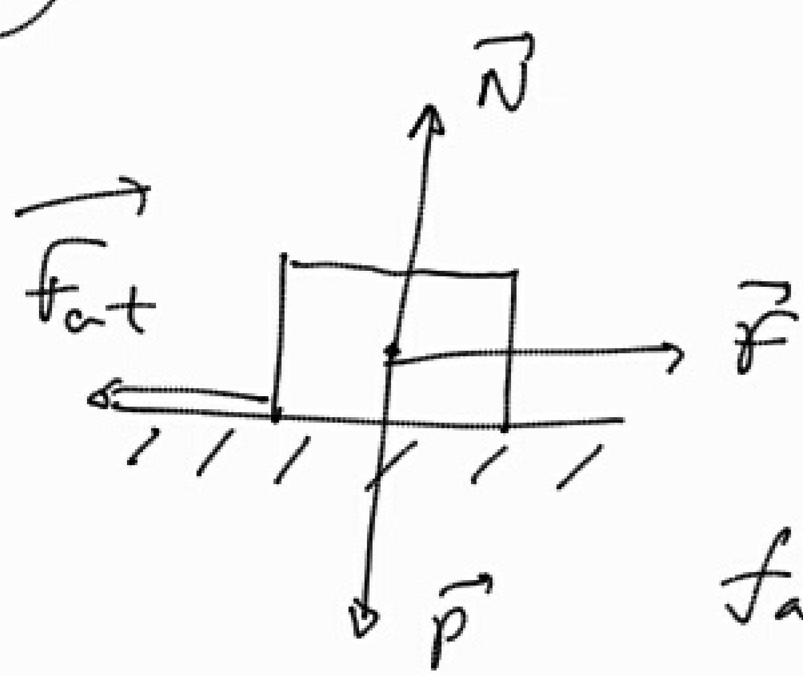

$P = mg = 4 \cdot 10$

$P = 40\,N$

$N = P \rightarrow N = 40\,N$

$f_{at\,max} = \mu \cdot N$

$f_{at\,max} = 0{,}5 \cdot 40 = 20\,N$

a.) $F < f_{at\,max} \rightarrow$ REPOUSO

$f_{AT} = F \rightarrow$ $f_{AT} = 10\,N$ e $a = 0$

b.) $F > f_{AT\,MAX}$

$f_{AT} = f_{AT\,MAX} = 20\,N$

$F - F_{at} = m \cdot a \Rightarrow 30 - 20 = 4a$

$4a = 10 \rightarrow$ $a = 2{,}5\,m/s^2$

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(6) $x = 4\cos\left(\frac{1}{2}\pi t + \pi\right)$

a.) $A = 4\,m$

$\omega = \frac{\pi}{2}\,rad/s$

$\varphi_0 = \pi\,rad$

b.) $\omega = 2\pi/T$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi/2} =$ $4\,s$

$f = \frac{1}{T} \rightarrow f = \frac{1}{4} =$ $0{,}25\,Hz$

c.) $v = \dfrac{dx}{dt} = \dfrac{d}{dt}\left[4\cos\left(\tfrac{1}{2}\pi t + \pi\right)\right]$

$v = -4 \cdot \dfrac{1}{2}\pi \operatorname{sen}\left(\dfrac{1}{2}\pi t + \pi\right)$

$v = -2\pi \operatorname{sen}\left(\dfrac{1}{2}\pi t + \pi\right)$

d.) $a = \dfrac{dv}{dt} = \dfrac{d}{dt}\left[-2\pi \operatorname{sen}\left(\dfrac{1}{2}\pi t + \pi\right)\right]$

$= -2\pi \cdot \dfrac{1}{2} \cdot \pi \cos\left(\dfrac{1}{2}\pi t + \pi\right)$

$= -\pi^2 \cos\left(\dfrac{1}{2}\pi t + \pi\right)$

e.)

$\operatorname{sen}\left(\dfrac{1}{2}\pi t + \pi\right) = 1$

$|v|_{MAX} = |-2\pi| = 2\pi\ m/s$

$\cos\left(\dfrac{1}{2}\pi t + \pi\right) = 1$

$|a|_{MAX} = \pi^2\ m/s^2$

(7) $P_c = E$

$P_c = m_c \cdot g$

$m_c = d_c \cdot V_c$

$V_c = A \cdot h$

$\Rightarrow P_c = d_c \cdot A \cdot h \cdot g$

$E = d_e \cdot V_e \cdot g$

$V_e = V_{imerso}$

$V_i = A \cdot h_{imerso}$

$V_{imerso} = A \cdot \frac{3}{4} h$

$\therefore \boxed{E = d_e \cdot A \cdot \frac{3}{4} \cdot h \cdot g}$

$P_a = E$

$d_c \cdot \cancel{A} \cdot \cancel{h} \cdot \cancel{g} = d_e \cdot \cancel{A} \cdot \frac{3}{4} \cdot \cancel{h} \cdot \cancel{g}$

$d_c = \frac{3}{4} d_e \Rightarrow \frac{d_c}{d_e} = \frac{3}{4}$

$d_{c,e} = \frac{3}{4}$

(8) a.) $\boxed{\lambda = 4m}$; b.) $\boxed{A = 10m}$

c.) $f = \frac{v}{\lambda} = \frac{200}{4} = \boxed{50 Hz}$

d.) $T = \frac{1}{f} = \frac{1}{50} = \boxed{0,021}$

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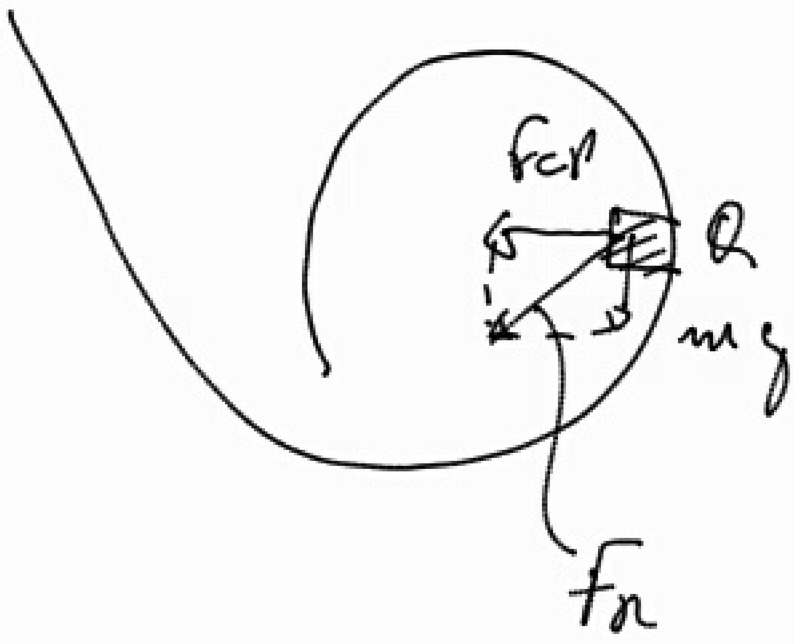
(9) a.) USANDO A CONSERVAÇÃO DA ENERGIA:

$$mg(5R) = mgR + \frac{1}{2} m V_Q^2$$

$$\cancel{m}g(4R) = \frac{1}{2} \cancel{m} V_Q^2 \therefore V_Q^2 = 8gR$$

$$\therefore V_Q = \sqrt{8gR}$$

CÁLCULO DA FORÇA RESULTANTE:



$$F_n^2 = F_{cp}^2 + m^2 g^2$$

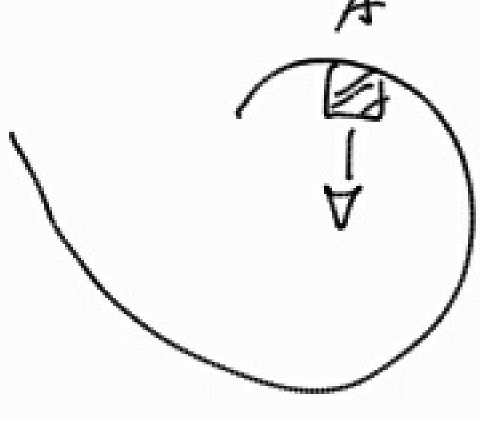
$$F_n^2 = \left(\frac{m V_Q^2}{R}\right)^2 + m^2 g^2 =$$

$$= \left(\frac{m}{R} \cdot 8gR\right)^2 + m^2 g^2$$

$$= 64 m^2 g^2 + m^2 g^2$$

$$F_n = \sqrt{65}\, mg$$

b.)



$$\frac{m V_A^2}{R} = F_{cp} = mg + N$$

NO MOMENTO QUE

DESPRENDE-SE $N = 0$

$$m \frac{V_A^2}{R} = mg$$

DA CONSERVAÇÃO DA ENERGIA :

$$mgy = 2mgR + \frac{1}{2} m V_A^2$$

MAS, $V_A^2 = \frac{R \cancel{m} g}{\cancel{m}} \rightarrow V_A^2 = Rg$

$$\Rightarrow \cancel{m}\cancel{g}y = 2\cancel{m}\cancel{g}R + \frac{1}{2}\cancel{m}R\cancel{g}$$

$$y = 2R + \frac{R}{2} \therefore y = \frac{5}{2}R$$