UNILATERAL FIN LINE DIRECTIONAL COUPLER

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Abstract

Photonic band gap (PBG) crystals have been used as a perfectly reflecting substrate for many millimeter wave applications. In this work the fin line directional coupler with substrate PBG was analyzed using the TTL – Transverse Transmission Line – method. Compared to other full wave methods the TTL is an efficient tool to determine the coupler characteristics, making possible a significant algebraic simplification of the equations involved in the process. In order to analyze this structure the effective dielectric constant, the attenuation constant and the coupling were determined. The results obtained for this application and the conclusions are presented.

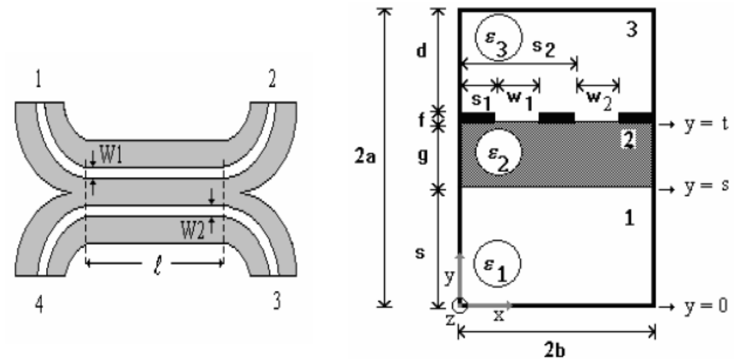
Key Words – Fin Lines, TTL – Transverse Transmission Line, PBG – Photonic Band Gap, Directional Coupler.

I. INTRODUCTION

Photonic band gap crystals have emerged as a new class of periodic dielectric structures where electromagnetic waves propagation is forbidden for all frequencies in the photonic band gap [1]. This material has a periodic arrangement of cylinders immersed in air with diameters and spacing of less than a wave length [2]-[4]. This substrate can improve the band width and eliminate the propagation of undesirable modes.

Many integrated circuits for millimeter wave applications can be made using fin line techniques, including, beyond the fin line circuit, circuits inserted in metal and other standards circuits, mounted in the wave guides E-plane [5].

This letter demonstrates an application of the 2D layer-by-layer PBG crystal: an efficient unilateral fin line directional coupler. The directional coupler can be made by the use of he natural coupling between the two symmetric located fins. The analysis is made using the TTL method and the coupling definitions. Fig. 1 shows this coupler.



(a) (b)

Fig. 1 – (a) Fin line directional coupler superior interior view; (b) unilateral fin line with coupled slots and thin conductors transversal section.

The coupled unilateral fin lines consist of a rectangular wave guide with three dielectric regions. Fig. 2 shows the structure in perspective. The even and odd modes are considered in the analysis.

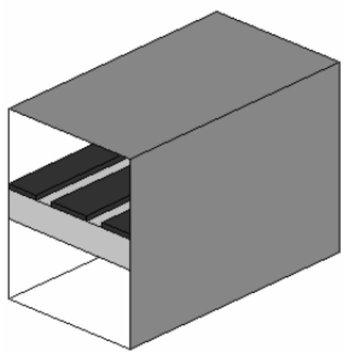


Fig. 2 – Coupled Unilateral Fin Line three dimensional view.

II. DEVELOPMENT

From the Maxwell's equations, the electromagnetic fields are developed:

$$\nabla \times \vec{E} = -j\omega\mu\vec{H} \qquad (1) \qquad \nabla \times \vec{H} = j\omega\varepsilon\vec{E} \qquad (2)$$

Separating the x and z components the final equations in the Fourier Transform Domain considering the i^{th} regions are obtained:

$$\tilde{E}_{xi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\alpha_n\frac{\partial}{\partial y}\tilde{E}_{yi} - j\omega\mu\Gamma\tilde{H}_{yi}\right] \quad (3) \qquad \tilde{E}_{zi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-\Gamma\frac{\partial}{\partial y}\tilde{E}_{yi} - \omega\mu\alpha_n\tilde{H}_{yi}\right] \quad (4)$$

$$\tilde{H}_{xi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\alpha_n\frac{\partial}{\partial y}\tilde{H}_{yi} + j\omega\varepsilon\Gamma\tilde{E}_{yi}\right] \quad (5) \qquad \tilde{H}_{zi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-\Gamma\frac{\partial}{\partial y}\tilde{H}_{yi} + \omega\varepsilon\alpha_n\tilde{E}_{yi}\right] \quad (6)$$

Where:

$\gamma_i^2 = \alpha_n^2 - \Gamma^2 - k_i^2$, is the propagation constant in "y" direction; α_n is the spectral variable in "x" direction.

$k_i^2 = \omega^2\mu\varepsilon = k_0^2\varepsilon_{ri}^*$ is the wave number of the ith dielectric region;

$\varepsilon_{ri}^* = \varepsilon_{ri} - j\dfrac{\sigma_i}{\omega\varepsilon_0}$ is the material relative dielectric constant with losses;

$\varepsilon_i = \varepsilon_{ri}^* \cdot \varepsilon_0$ is the dielectric constant of the ith region;
$\Gamma = \alpha + j\beta$ is the complex propagation constant; and,
$\omega = \omega_r + j\omega_i$ is the complex angular frequency.

A. Fields Structures

The solutions of the fields' equations for the three regions in study are given by:
For region 1:

$$\tilde{E}_{y1} = A_{1e} \cdot \cosh\gamma_1 y \quad (7) \qquad \tilde{H}_{y1} = A_{1h} \cdot \cosh\gamma_1 y \quad (8)$$

For region 2:

$$\tilde{E}_{y2} = A_{2e} \cdot \sinh\gamma_2 y + B_{2e} \cdot \cosh\gamma_2 y \quad (9)$$

$$\tilde{H}_{y2} = A_{2h} \cdot \sinh\gamma_2 y + B_{2h} \cdot \cosh\gamma_2 y \quad (10)$$

For region 3:

$$\tilde{E}_{y3} = A_{3e} \cdot \cosh\gamma_3(da - y) \quad (11) \qquad \tilde{H}_{y3} = A_{3h} \cdot \cosh\gamma_3(da - y) \quad (12)$$

For the determination of the unknown constants described above the boundary conditions are applied in the structure shown in Fig. 1b, in y=s and y=t. The electromagnetic fields general equations as function of $\tilde{E}_{xt}$ and $\tilde{E}_{zt}$, which are the tangential components of the electric fields are obtained.

B. The Admittance Matrix

To calculate the propagation constant, the magnetic boundary conditions are applied.

$$\tilde{H}_{x2} - \tilde{H}_{x3} = \tilde{J}_{zt} \quad (13) \qquad \tilde{H}_{z2} - \tilde{H}_{z3} = -\tilde{J}_{xt} \quad (14)$$

were $\tilde{J}_{xt}$ and $\tilde{J}_{zt}$ are the electric current densities in the fins.

Substituting the magnetic fields equations in the above equations and isolating the terms in the electric fields, were find the equations that can be written through the admittance functions:

$$Y_{xx}\tilde{E}_{xt} + Y_{xz}\tilde{E}_{zt} = \tilde{J}_{xt} \quad (15) \qquad Y_{zx}\tilde{E}_{xt} + Y_{zz}\tilde{E}_{zt} = \tilde{J}_{zt} \quad (16)$$

This system can be written in the matrix form:

$$\begin{bmatrix} Y_{xx} & Y_{xz} \\ Y_{zx} & Y_{zz} \end{bmatrix} \begin{bmatrix} \tilde{E}_{xt} \\ \tilde{E}_{zt} \end{bmatrix} = \begin{bmatrix} \tilde{J}_{xt} \\ \tilde{J}_{zt} \end{bmatrix} \qquad (17)$$

C. Fields Expansions in terms of Base Functions

The $\tilde{E}_{xt}$ and $\tilde{E}_{zt}$ fields are expanded in terms of base functions:

$$\tilde{E}_{xt} = \sum_{i=1}^{n} a_{xi} \cdot \tilde{f}_{xi} \qquad (18) \qquad \tilde{E}_{zt} = \sum_{j=1}^{m} a_{zj} \cdot \tilde{f}_{zj} \qquad (19)$$

where n and m are positive integers.

As the fields have the contribution of two slots, they can be written as:

$$\tilde{E}_{xt} = \tilde{E}_{xt1} + \tilde{E}_{xt2} \qquad (20) \qquad \tilde{E}_{zt} = \tilde{E}_{zt1} + \tilde{E}_{zt2} \qquad (21)$$

where the 1 and 2 indexes are related to the first and second slots.

D. Propagation Constant

Continuing, the scalar product of equation (17) is made by the set of base functions according to the Galerkin method, the particular case method of moments. With this, the current densities turn out zero and a new matrix equation is obtained.

$$\begin{bmatrix} K_{xx}^{11} & K_{xx}^{12} & K_{xz}^{11} & K_{xz}^{12} \\ K_{xx}^{21} & K_{xx}^{22} & K_{xz}^{21} & K_{xz}^{22} \\ K_{zx}^{11} & K_{zx}^{12} & K_{zz}^{11} & K_{zz}^{12} \\ K_{zx}^{21} & K_{zx}^{22} & K_{zz}^{21} & K_{zz}^{22} \end{bmatrix} \begin{bmatrix} a_{1x} \\ a_{2x} \\ a_{1z} \\ a_{2z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (22)$$

Where for example:

$$K_{xx}^{1j} = \sum_{-\infty}^{\infty} Y_{xx} \tilde{f}_{jx} \cdot \tilde{f}_{1x}^{\,*} \qquad (23)$$

So that, for the system (22) to have a non trivial solution, the matrix determinant of the “K” coefficients must be equal to zero. This determinant is represented by a transcendental equation that has as roots the attenuation constant “α” and phase constant “β”, so we have the complex propagation constant $\Gamma = \alpha + j\beta$.

Finally once the propagation constant is obtained the effective dielectric permittivity “ε_{eff}” in the structure is given by:

$$\varepsilon_{eff} = \left(\frac{\beta}{k_0} \right)^2 \qquad (24)$$

III. DIRECTIONAL COUPLER

The directional coupler offers direct coupling. In this system when the signal arrives in port 1, the port 3 will be coupled and port 4 will be isolated. The even and odd modes propagate with different phase velocities, and the coupling is periodic through the line length.

The L length to total power transference from port 1 to port 3 is [7].

$$L = \pi / (\beta_{even} - \beta_{odd}) \quad (25)$$

where β_{even} and β_{odd} are phase constants for the even and odd modes, respectively, this constants are calculated by the use of the TTL method.

The amplitude of the coupling coefficient between ports 1 and 3, is.

$$|S_{13}| = \sin(\pi/2 \cdot l/L) \quad (26)$$

and the coupling amplitude coefficient between ports 1 and 2 is,

$$|S_{12}| = \cos(\pi/2 \cdot l/L \quad (27)$$

where the length l is calculated with equation (25)

The expression for the coupling is defined as [7]:

$$C_3 = 20.\log\left(\frac{1}{|S_{13}|}\right) \quad (28)$$

IV. PBG STRUCTURE

For a non-homogeneous structure, the incident wave undergoes a process of multiple scattering. A solution can be obtained through a numerical process called homogenization [2-5]. The process is based on the theory related to the diffraction of an incident electromagnetic plane wave imposed by the presence of air immersed cylinders in a homogeneous material.

In the homogenization [5] process the bidimensional medium is reduced to a one-dimensional medium. To do so, let us imagine that the crystal is made of a stack of diffraction gratings (what we now deal with is an infinite structure in the *x* direction). Numerically, we observe that the last gap corresponds to a range of wavelengths that give rise to only one diffraction order for such gratings. Consequently, forgetting the evanescent waves leads to the conclusion that each grating behaves as a homogeneous slab.

The problem is to specify an effective index for this slab. In fact, what is done consists in ‘‘homogenizing’’ the crystal slice by slice, as depicted in Fig. 3. It should be noted here that the wavelength is not very large with respect to the size of the scatterers, meaning that, properly speaking, we are not in the rigorous framework of homogenization theory. Nevertheless, let us try to apply the homogenization rules so as to get an effective index. The rods with permittivity ε_1 are embedded in a medium of permittivity ε_2. The procedure consists in dividing the structure into a superposition of homogenized layers. The layers containing the rods are broken up into cells hose *y* size (resp. *x* size) is the diameter of a rod (2*r*) (resp. the period *d*).

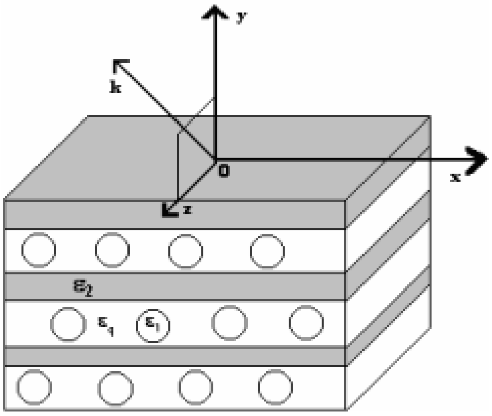


Fig. 3 – Homogenized bidimensional PBG crystal.

According to homogenization theory the effective permittivity depends on the polarization [5]. For the *s* and *p* polarization, respectively, we have:

$$\varepsilon_{eq} = \beta(\varepsilon_1 - \varepsilon_2) + \varepsilon_2 \quad (29)$$

$$\frac{1}{\varepsilon_{eq}} = \frac{1}{\varepsilon_1}\left\{1 - \frac{3\beta}{\dfrac{2/\varepsilon_1 + 1/\varepsilon_2}{1/\varepsilon_1 - 1/\varepsilon_2} + \beta - \dfrac{\alpha(1/\varepsilon_1 - 1/\varepsilon_2)}{4/3\varepsilon_1 + 1/\varepsilon_2}\beta^{10/3} + O(\beta^{14/3})}\right\} \quad (30)$$

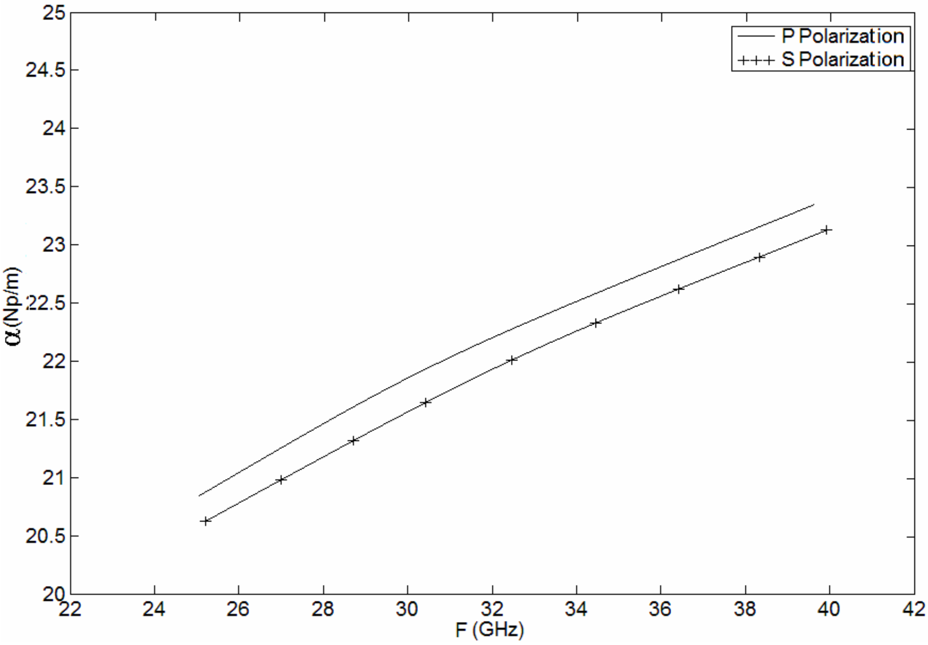
β is defined as the ratio between the area of the cylinders and the area of the cells, α is an independent parameter whose value is equal to 0.523.

The preceding way of homogenizing the photonic crystal involves choosing a particular direction, namely the one that is orthogonal to the direction of the incident wave vector. The results obtained are expected to hold for incidence angles near the normal direction, because the bandgaps do not change notably for small variation of the incidence angle [5].

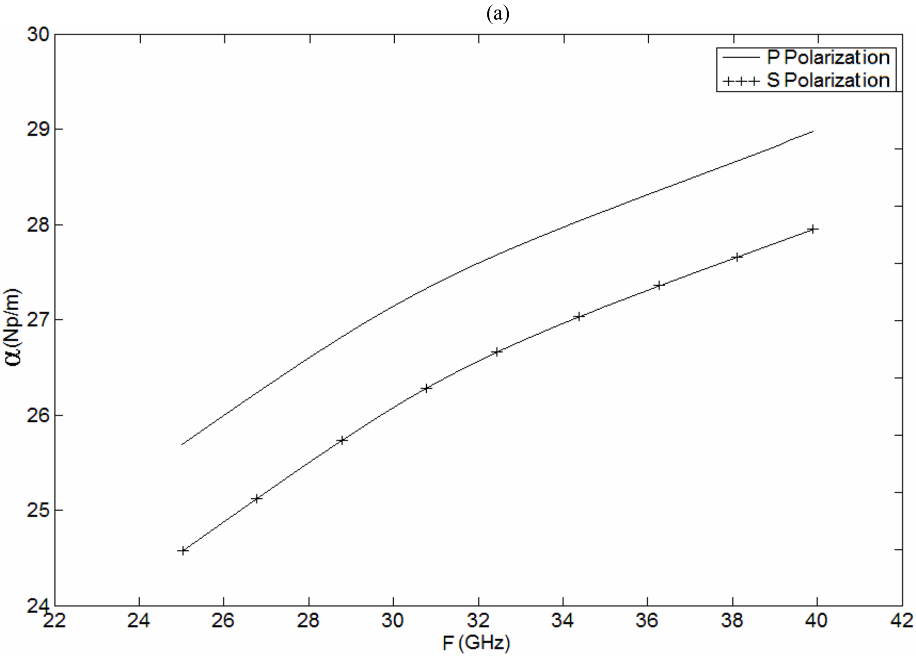
V. Numerical Results

In order to calculate the numerical results it was developed a computational program in Fortran PowerStation language, according to the previous theoretical analyses. In this work results for the phase constant, effective dielectric permittivity, for the even and odd modes, and coupling are shown.

The new results are related to a unilateral fin line with PBG substrate, with two coupled slots, in a WR-28 millimeter wave guide with 2g = 0.254 mm, s = 3.302 mm (region 1 width), x1 = 1.078 mm, x2 = 2.278 mm, w1 = w2 = 0.2 mm, ε_{r2} = 8.7209 for the *p* polarization and ε_{r2}= 10.233 for the *s* polarization, s'=0.5 mm (half the distance between the slots). The figures show the results with an excellent convergence with the theoretical results, Fig.4 (a) show the attenuation for the even mode, and Fig. 4 (b) show the attenuation for the odd mode.



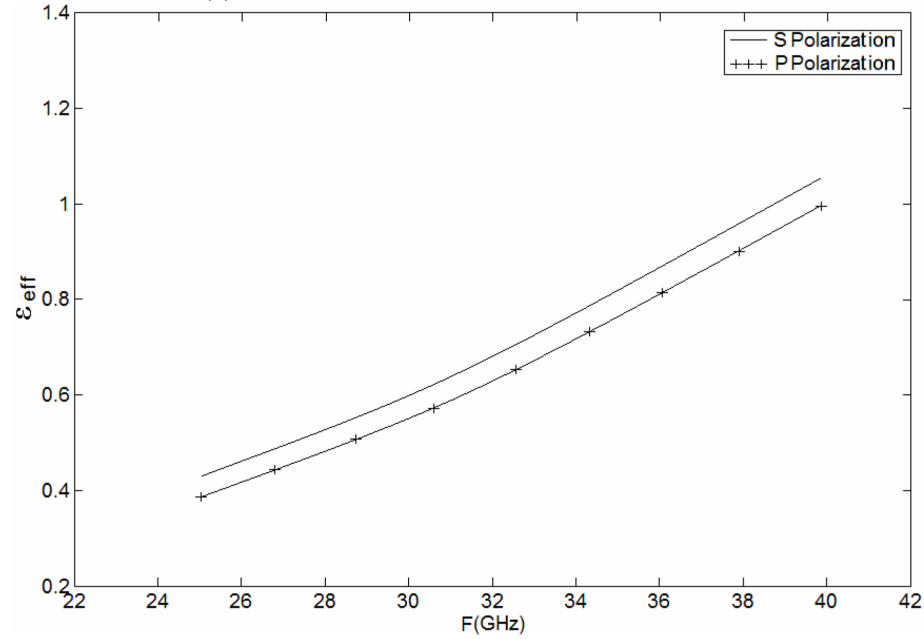
(a)



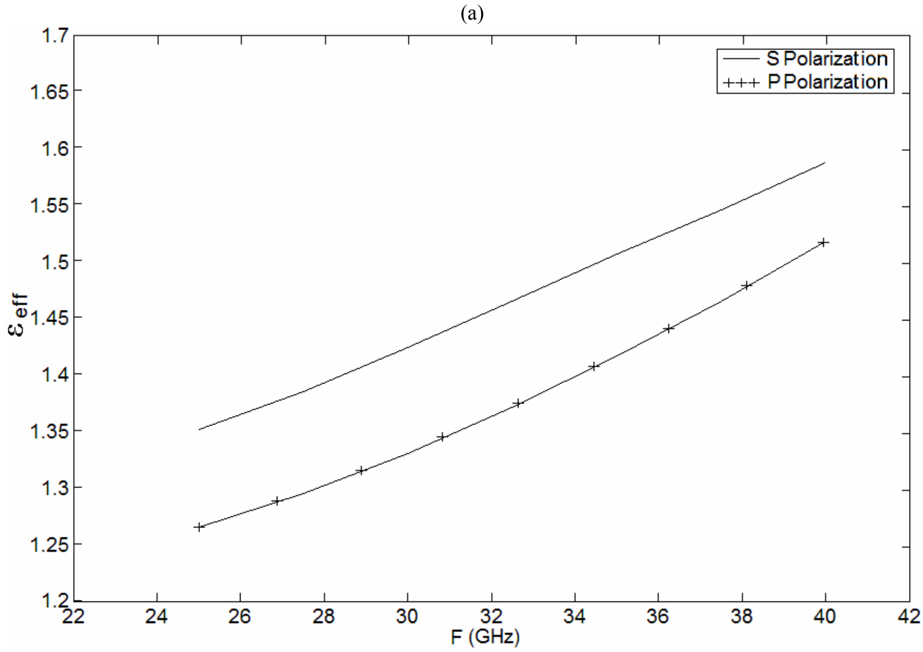
(b)

Fig. 4 – Attenuation as a function of the frequency, for a unilateral fin line with two coupled slots, with PBG substrate in a WR-28 millimeter wave guide, (a) for the even mode (b) for the odd mode.

Fig. 5 shows the effective dielectric permittivity as a function of the frequency, (a) for the even mode and (b) for the odd mode.



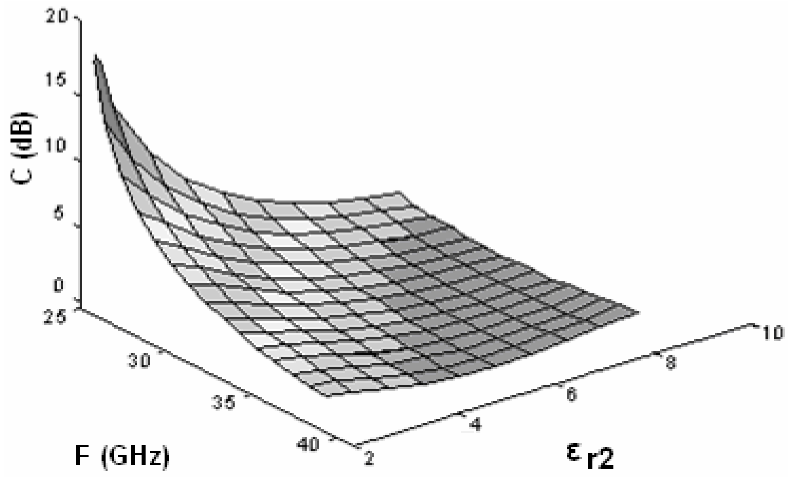
(a)



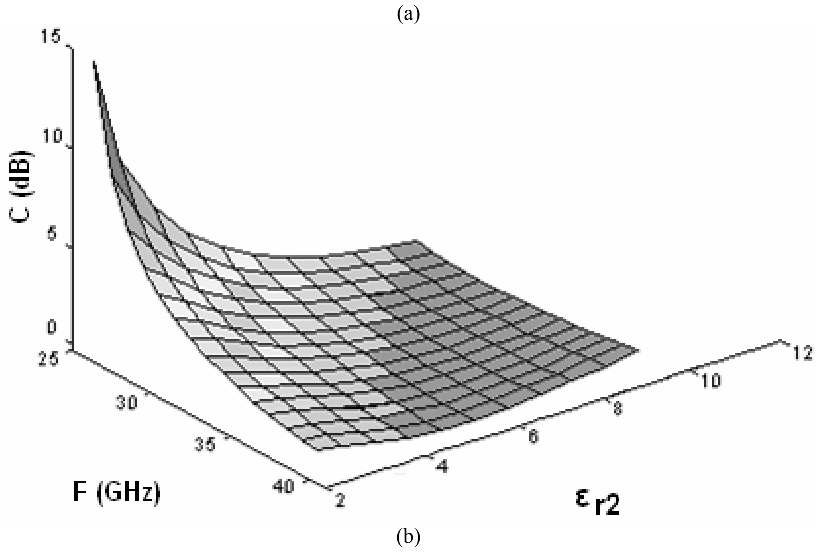
(b)

Fig. 5 – Effective dielectric permittivity as a function of the frequency for a unilateral fin line, with two coupled slots, with PBG substrate in a WR-28 millimeter wave guide, (a) for the even mode (b) for the odd mode.

Results for the coupling in port 3, as a function of the frequency and the dielectric constant in region 2, as shown in Fig. 6 for the unilateral fin line directional coupler. For this results is considered a WR-28 millimeter wave guide, with $w_1 = w_1 = 0.2$ mm, $l = 0.5$ mm, $g = 0.254$ mm and $s = 3.302$ mm. $2s'=0.789$ (s' = distance form a slot to structure the center) and conductivity, $\sigma_2 = 1.0$, relative permittivity of region 2, Fig. 6 (a) $\varepsilon e_2 = 8.7209$ P polarization and Fig. 6 (b) $\varepsilon r2 = 10.233$ S polarization.



(a)



(b)

Fig. 6 – Port 3 coupling as a function of the frequency and region 2 permittivity for, a unilateral fin line directional coupler (a) P polarization, (b) S polarization.

VI. CONCLUSIONS

Theoretical and numerical results have been presented for the unilateral fin line directional coupler with 2D PBG photonic substrate. The TTL method, in the Fourier transform domain was used in the determination of the electrical and magnetic fields, in all regions.

Results in 2D and 3D were obtained for the propagation constant, including the attenuation and phase constant, coupling in port 3 and effective dielectric constant for the fin line structure. It was observed a considerable reduction in the algebraic development using the TTL method

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